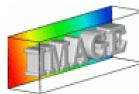


ROBUST VARIATIONAL ESTIMATION OF PDF FUNCTIONS FROM DIFFUSION MR SIGNAL

Haz-Edine Assemlal, David Tschumperlé, Luc Brun

GREYC IMAGE (CNRS UMR 6072), CAEN, FRANCE

CDMRI 2008



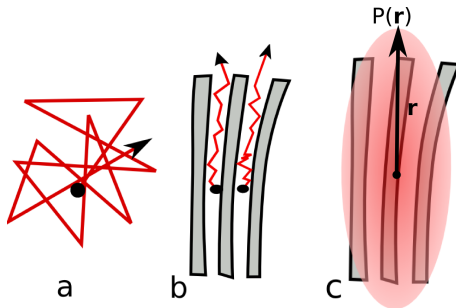
① ESTIMATION OF DTI, ODF

② OUR CONTRIBUTION

- Variational estimation of PDF-Characteristics
- Robust Rician estimation & regularization

③ RESULTS

DIFFUSION MRI: BROWNIAN MOTION

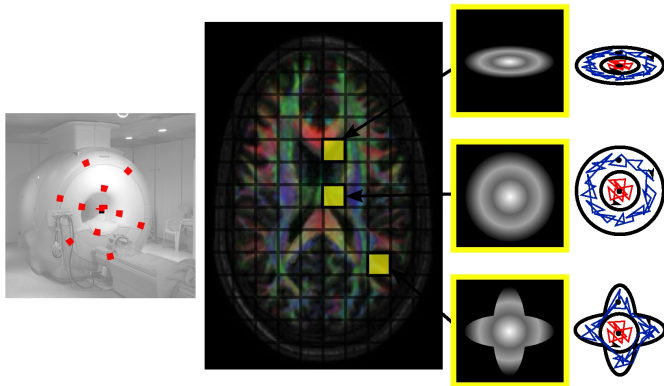


- a: Brownian motion of water molecules
- b: Constraints due to body internal structures
- c: Signal = Local macroscopic diffusion

OBJECTIVE

To estimate the probability P of the macroscopic displacement $\mathbf{p}$ during a diffusion time dt .

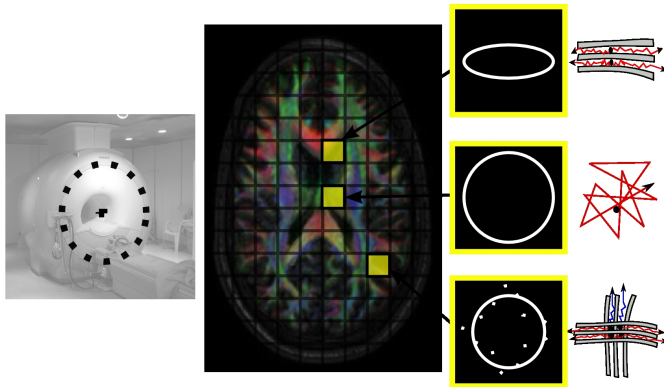
SIGNAL ACQUISITION



(a) PDF

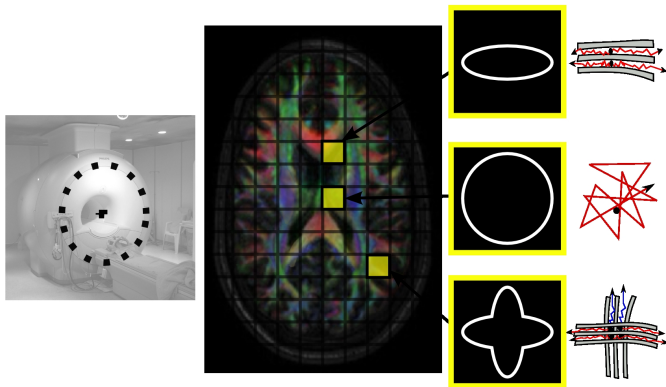
FIGURE: Acquisition steps.

SIGNAL ACQUISITION



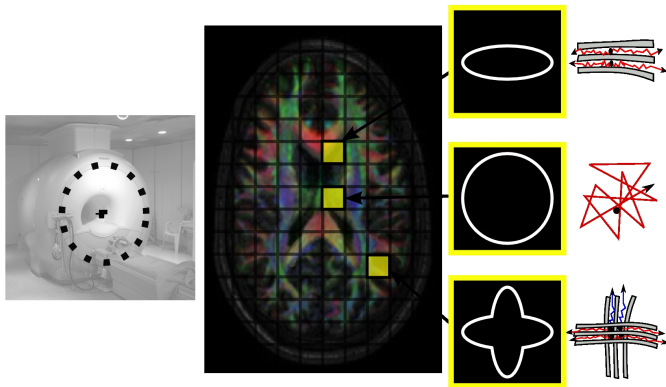
(a) DTI (Basser94)

FIGURE: Acquisition steps.



(a) ODF (Tuch02)

FIGURE: Acquisition steps.



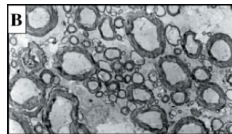
(a) ODF (Tuch02)

FIGURE: Acquisition steps.

Why measuring PDF ?

INTEREST OF RADIAL PDF

- Information on cells micro-structure that composed the organic tissue. Ex: axon diameter, number of compartments.
- May increase detection of anomalies such as demyelination, a symptom of multiple sclerosis.



Spinal cord
[Cohen02]

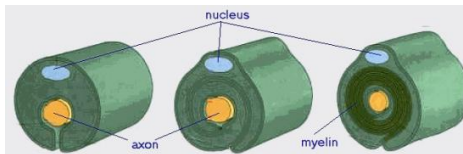


FIGURE: Myelination of an axone [www.jdaross.cwc.net]

DIFFUSION IN A BI-HOMOGENEOUS ENVIRONMENT

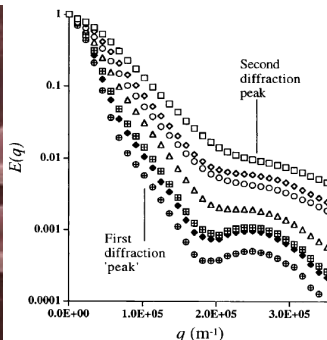
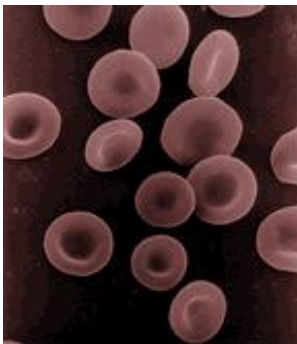


FIGURE: Experimental graph: human erythrocytes rate for decreasing values of hematocrites. [Kuchel97]

Empirical approximation of signal by a bi-exponential function (compartments: intra/extracellular diffusion).

DIFFUSION IN A COMPLEX ENVIRONMENT

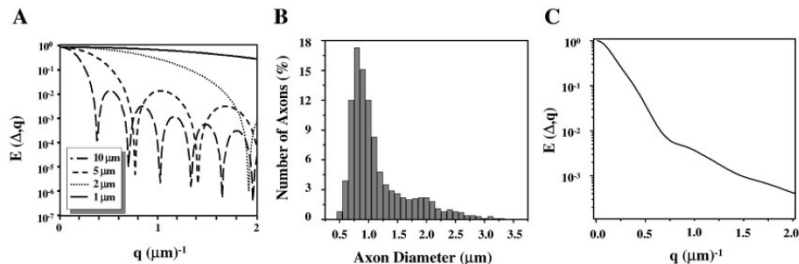


FIGURE: Simulation plot: fibers set of various diameters. [Cohen02]

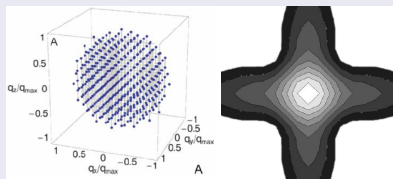
OBSERVATIONS

Important information are found in the *radial* diffusion profile.

HIGH ANGULAR RESOLUTION DIFFUSION IMAGING

$$P = \mathcal{F}[E], \text{ i.e. } P(\mathbf{p}) = \int_{\mathbf{q}} E(\mathbf{q}) e^{-2\pi i \mathbf{q} \mathbf{p}} d\mathbf{q} \text{ [Callaghan91]}$$

DSI: FOURIER TRANSFORM [WEDEEN00]

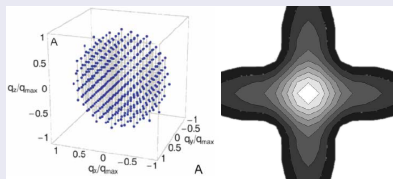


- Very long acquisition time
- Needs high gradients $\Rightarrow$ Magnetic field distortion

HIGH ANGULAR RESOLUTION DIFFUSION IMAGING

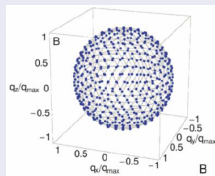
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DSI: FOURIER TRANSFORM [WEDEEN00]



- Very long acquisition time
- Needs high gradients $\Rightarrow$ Magnetic field distortion

QBI: FUNK-RADON TRANSFORM [TUCH04]



- Radial projection of the PDF
- Orientation Density Function (ODF)

HARDI EXTENSION

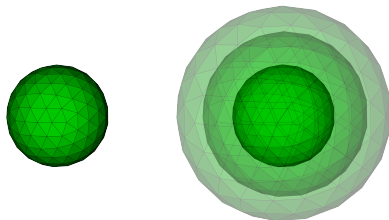


FIGURE: Example of HARDI extension [Assaf05, Özarslan06, Wu07, Assemlal-et.al08].

Better distribution of samples on the Q -Space.

PROBLEM

Still insufficient number of samples for a Fourier transform. Which mathematical tool for the signal estimation ?

CONTINUOUS REPRESENTATION OF THE SIGNAL

Work presented at MICCAI 2008 [Assemlal08-et al].
Reconstruction with limited number of samples.

Continuous representation of the MR signal E in the following basis
(Spherical Polar Fourier SPF):

$$E(\mathbf{q}) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{nlm} R_n(\|\mathbf{q}\|) y_l^m \left(\frac{\mathbf{q}}{\|\mathbf{q}\|} \right) \quad (1)$$

where a_{nlm} expansion coefficients, R_n and y_l^m are respectively are radial and angular atoms.

The basis is *orthonormal* in spherical coordinates:

$$\int_{\mathbf{q} \in \mathbb{R}^3} \left[R_n(\|\mathbf{q}\|) y_l^m \left(\frac{\mathbf{q}}{\|\mathbf{q}\|} \right) \right] \cdot \left[R_{n'}(\|\mathbf{q}\|) y_{l'}^{m'} \left(\frac{\mathbf{q}}{\|\mathbf{q}\|} \right) \right] d\mathbf{q} = \delta_{nn'} \delta_{ll'} \delta_{mm'} \quad (2)$$

RADIAL ATOMS

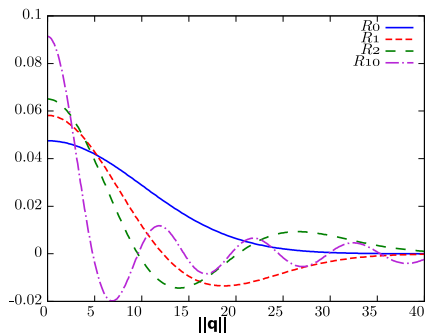


FIGURE: Some radial atoms R_n ,
 $\gamma = 100$

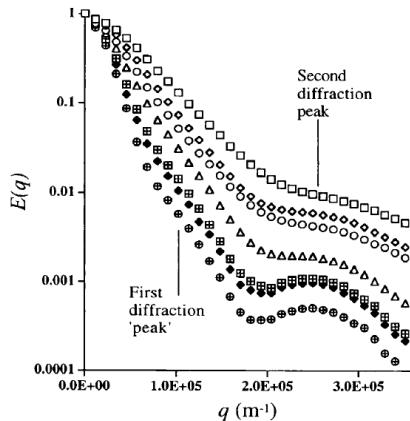


FIGURE: Experimental plot
 [Kuchel97]

$\mathcal{G}$	ODF	FRT	ISO
$h_{\mathbf{k}}(\mathbf{p})$	$\delta(1 - \frac{ \mathbf{p} \cdot \mathbf{k} }{\ \mathbf{p}\ \ \mathbf{k}\ })$	$J_0(2\pi \ \mathbf{q}'\ \ \mathbf{p}\)$	$\delta(\mathbf{k})$

TABLE: A non exhaustive list of PDF characteristics $\mathcal{G}$ and their associated projection function $h_{\mathbf{k}}$ at point $\mathbf{k}$.

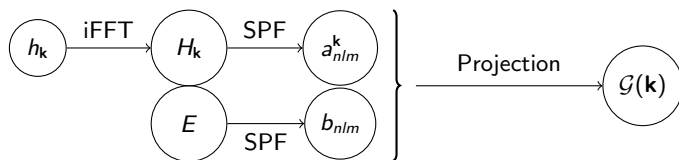


FIGURE: Overview of the algorithm for the computation of a PDF characteristic $\mathcal{G}$ at point $\mathbf{k}$

ALGORITHM

$\mathcal{G}$	ODF	FRT	ISO
$h_{\mathbf{k}}(\mathbf{p})$	$\delta(1 - \frac{ \mathbf{p} \cdot \mathbf{k} }{\ \mathbf{p}\ \ \mathbf{k}\ })$	$J_0(2\pi \ \mathbf{q}'\ \ \mathbf{p}\)$	$\delta(\mathbf{k})$

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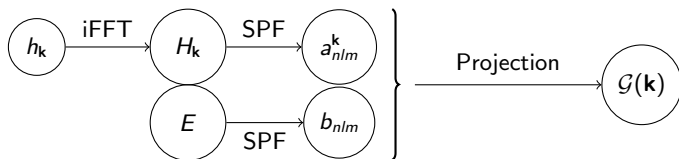


FIGURE: Overview of the algorithm for the computation of a PDF characteristic $\mathcal{G}$ at point $\mathbf{k}$

How to robustly estimate SPF coefficients b_{nlm} from noisy MR signal ?

1 ESTIMATION OF DTI, ODF

2 OUR CONTRIBUTION

- Variational estimation of PDF-Characteristics
- Robust Rician estimation & and regularization

3 RESULTS

ROBUSTNESS TO NOISE

PROBLEM

Acquisition noise not Gaussian but Rician.

Nonetheless most methods use least square estimation !

⇒ Bias especially at low SNR values (high b values).

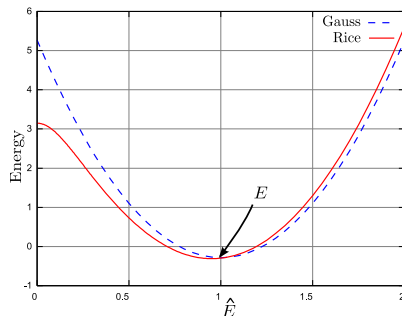


FIGURE: Energy associated to respectively Gaussian and Rician likelihood ψ functions. $E = 1$ and $\sigma = 0.5$.

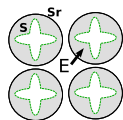
VARIATIONAL FRAMEWORK

Robustly estimate and regularize the SPF coefficients by minimizing the functional energy:

$$\min_A \left\{ \int_{\Omega_E} \left[\sum_k^{n_s} \psi(\hat{E}_k) \right] + \alpha_r \varphi(\|\nabla A\|) d\Omega_E \right\}, \text{ with } \hat{E} = M.A \quad (3)$$

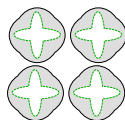
The best fitting coefficients A are computed with a gradient descent coming from the Euler-Lagrange derivation of the energy. This leads to a set of *multi-valued* partial derivate equation.

$$\begin{cases} A_{t=0} = U_0 \\ \frac{\partial A_j}{\partial t} = \sum_k^{n_s} M_{k,j} \psi'(\hat{E}_k) + \alpha_r \operatorname{div}(\varphi(\|\nabla A\|)) \end{cases} \quad (4)$$



iteration 0

(GREYC IMAGE)



iteration 1



...



iteration n

CDMRI 2008

15 / 24

DIFFUSION MRI

ADVANTAGES: ADAPTIVE TO NOISE DISTRIBUTION

- ψ -likelihood function adapted to MRI noise law:

The best ψ function is the one specific to MR scanners, ie. Rice distribution:

$$p(E|\hat{E}, \sigma) = \frac{E}{\sigma^2} \exp\left(-\frac{(E^2 + \hat{E}^2)}{2\sigma^2}\right) I_0\left(\frac{E \cdot \hat{E}}{\sigma^2}\right) \quad (5)$$

We seek $\hat{E}$ which maximizes a posteriori (MAP) the log-posterior probability [Basu06]

$$\log p(\hat{E}|E) = \log p(E|\hat{E}) + \log p(\hat{E}) - \log p(E) \quad (6)$$

Consequently the pointwise likelihood is

$$\log p(E|\hat{E}, \sigma) = \log \frac{E}{\sigma^2} - \frac{(E^2 + \hat{E}^2)}{2\sigma^2} + \log I_0\left(\frac{E \cdot \hat{E}}{\sigma^2}\right) = \psi(\hat{E}) \quad (7)$$

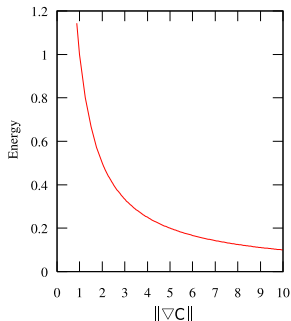
ADVANTAGES: REGULARITY

- Ensure a global regularity of the SPF field:

$$\varphi(\|\nabla \mathbf{A}\|) = \varphi(\sum_{nlm} \|\nabla A_{nlm}\|)$$

φ is a contour preserving function, widely used in image processing.

Example of possible regularization function φ



1 ESTIMATION OF DTI, ODF

2 OUR CONTRIBUTION

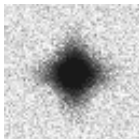
- Variational estimation of PDF-Characteristics
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3 RESULTS

RICIAN VS GAUSSIAN LIKELIHOOD FUNCTION



a) Truth



b) Noisy



c) Gaussian



d) Rician



$|Gauss - Truth|$



$|Rice - Truth|$

VALIDATION ON SYNTHETICAL DATA: LIKELIHOOD

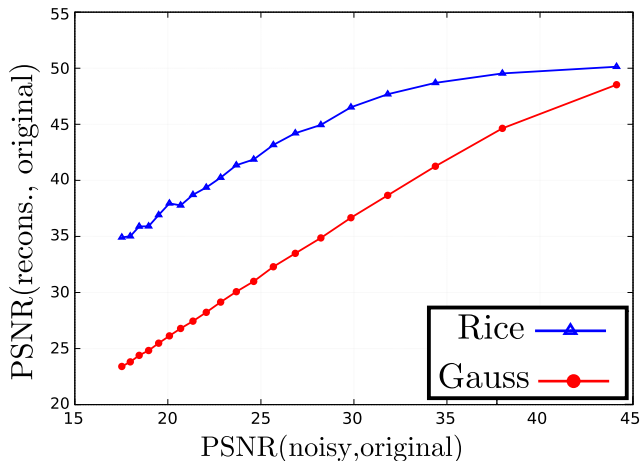


FIGURE: Synthetic phantom of networks of crossing fibers. Performances of likelihood functions on increasing levels of noise.

REGULARIZATION VS NO REGULARIZATION

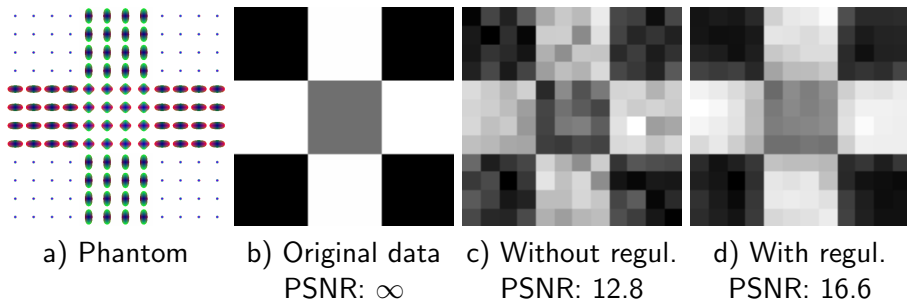
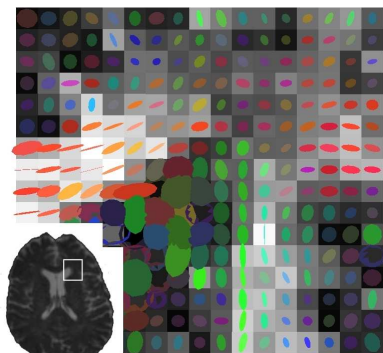


FIGURE: Effects of spatial regularization on the GFA. Isotropic area are black, anisotropic area are white. $\text{PSNR}(\text{noisy}, \text{original}) = 18.5$.

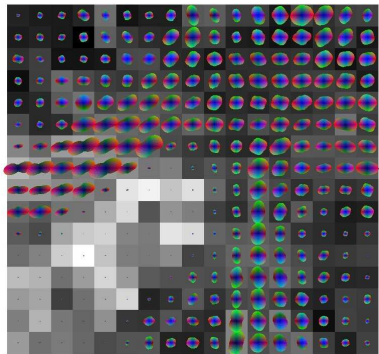
RESULTS OF *in-vivo* EXPERIENCES



(a) DTI (Basser94)

FIGURE: Brain white matter: ODF overlaid on a GFA map. DTI(a) and QBI(b) were computed with $b = 3000 \text{ s/mm}^2$. Our method (c) shows the obtained ODF with $b = 1000$ and 3000 s/mm^2 .

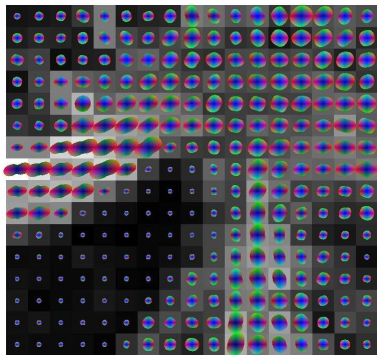
RESULTS OF *in-vivo* EXPERIENCES



(b) QBI (Descoteaux07)

FIGURE: Brain white matter: ODF overlayed on a GFA map. DTI(a) and QBI(b) were computed with $b = 3000 \text{ s/mm}^2$. Our method (c) shows the obtained ODF with $b = 1000$ and 3000 s/mm^2 .

RESULTS OF *in-vivo* EXPERIENCES



(c) Our method, $\mathcal{G}=\text{ODF}$

FIGURE: Brain white matter: ODF overlayed on a GFA map. DTI(a) and QBI(b) were computed with $b = 3000 \text{ s/mm}^2$. Our method (c) shows the obtained ODF with $b = 1000$ and 3000 s/mm^2 .

RESULTS OF *in-vivo* EXPERIENCES

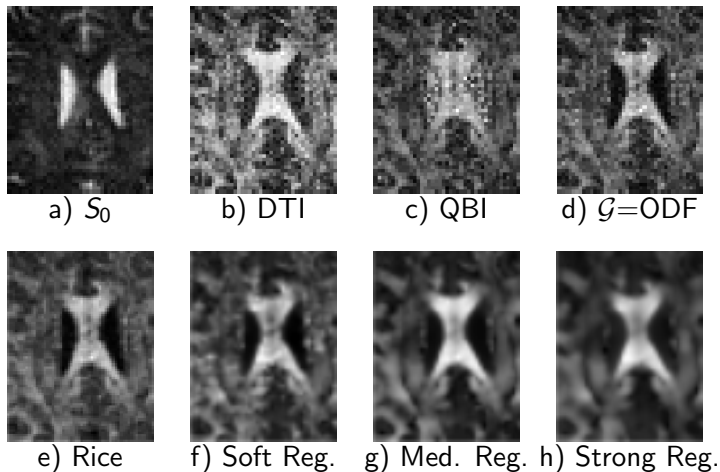


FIGURE: Comparison of GFA on region of corpus callosum and lateral ventricles.

- Variational framework for robust estimation of *various* characteristics on tissue microstructure from MR images.
- Rician estimation much improves the radial profile of MR Signal.
- Regularization enhances coherence and fibertracking may take advantage of this.

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Thank you for your attention.