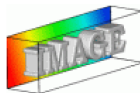


EFFICIENT COMPUTATION OF PDF-BASED CHARACTERISTICS FROM DIFFUSION MR SIGNAL

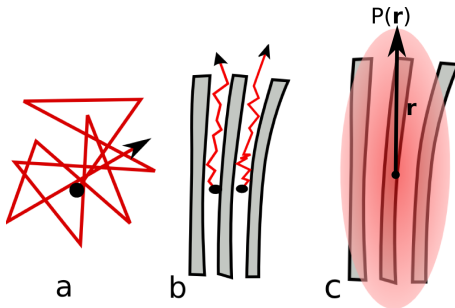
Haz-Edine Assemlal, David Tschumperlé, Luc Brun

GREYC IMAGE (CNRS UMR 6072)

MICCAI 2008



DIFFUSION MRI: BROWNIAN MOTION



- a: Brownian motion of water molecules
- b: Constraints due to body internal structures
- c: Signal = Local macroscopic diffusion

OBJECTIVE

To estimate the probability P of the macroscopic displacement $\mathbf{p}$ during a diffusion time dt .

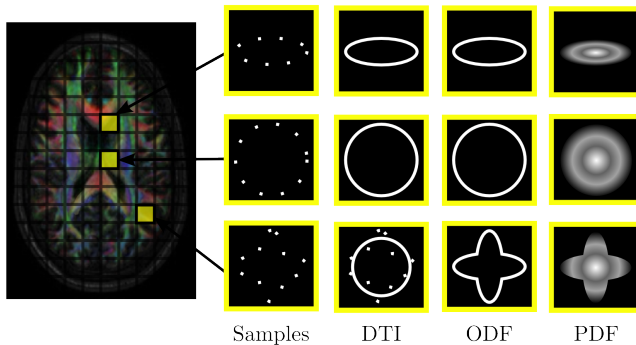
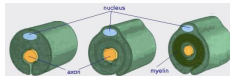
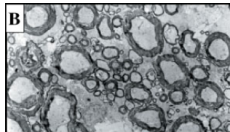


FIGURE: Acquisition steps.

Why measuring PDF ?

INTEREST OF RADIAL PDF

- Information on cells micro-structure that composed the organic tissue. Ex: axon diameter, number of compartments.
- May increase detection of anomalies such as demyelination, a symptom of multiple sclerosis.

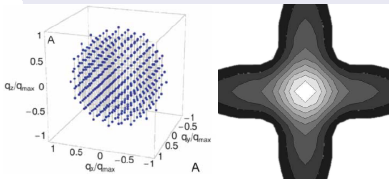


Why *don't* we estimate the radial part of the signal until now?

HIGH ANGULAR RESOLUTION DIFFUSION IMAGING

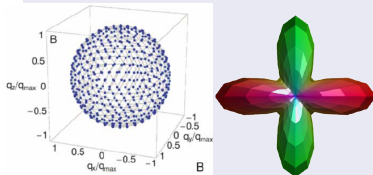
$$P = \mathcal{F}[E], \text{ i.e. } P(\mathbf{p}) = \int_{\mathbf{q}} E(\mathbf{q}) e^{-2\pi i \mathbf{q} \mathbf{p}} d\mathbf{q} \text{ [Callaghan91]}$$

DSI: FOURIER TRANSFORM [WEDEEN00]



- Very long acquisition time
- Needs high gradients $\Rightarrow$ Magnetic field distortion

QBI: FUNK-RADON TRANSFORM [TUCH04]



- Radial projection of the PDF
- Orientation Density Function (ODF)

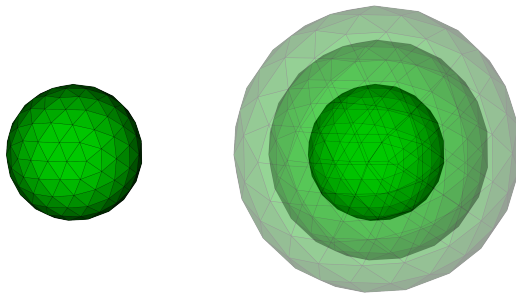


FIGURE: Example of HARDI extension [Assaf05, Özarslan06, Wu07, Asselmaj-etal08].

PROPOSITION

Better distribution of samples on the Q -Space.

MODEL FOR THE RADIAL DIFFUSION PROFILE

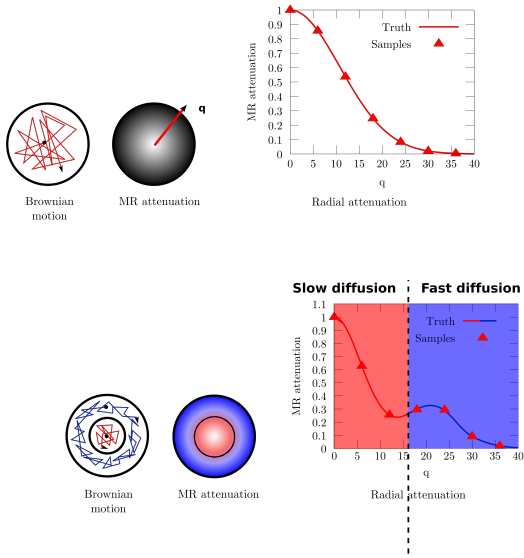


FIGURE: Mono vs Bi-exponential decay

DIFFUSION IN A HOMOGENEOUS ENVIRONMENT

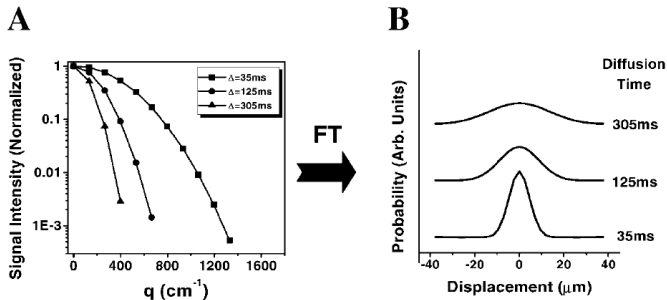


FIGURE: Experimental graph: MR signal of t-butanol at 25 °C and displacement probability depending on diffusion time. [Assaf00]

Gaussian decay in a homogeneous environment, results agree with macroscopic diffusion Fick's theory.

DIFFUSION IN A BI-HOMOGENEOUS ENVIRONMENT

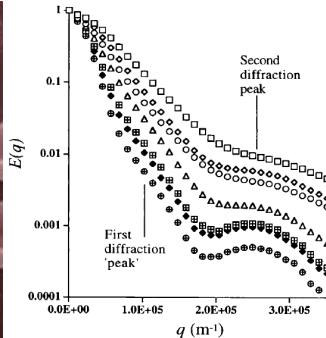
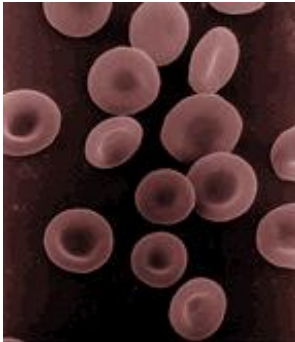


FIGURE: Experimental graph: human erythrocytes rate for decreasing values of hematocrites. [Kuchel97]

Empirical approximation of signal by a bi-exponential function (compartments: intra/extra diffusion).

DIFFUSION IN BRAIN

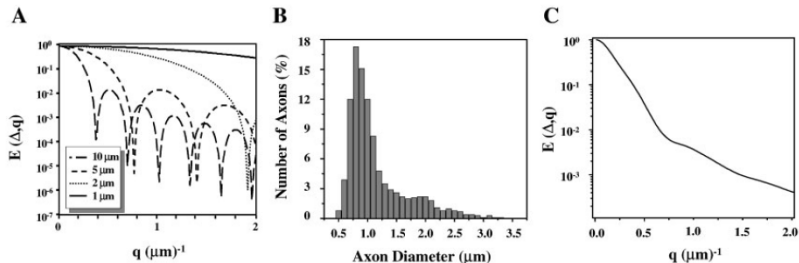
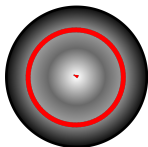


FIGURE: Simulation plot: fibers set of various diameters. [Cohen02]

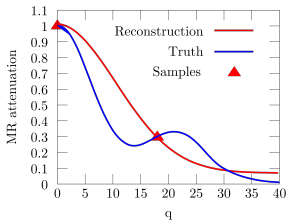
OBSERVATIONS

Important information are found in the *radial* diffusion profile.

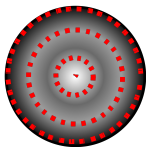
SAMPLING



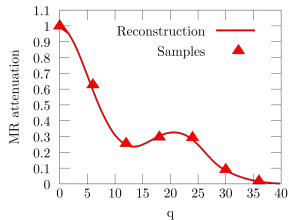
HARDI sampling



Reconstruction



HYDI sampling



Reconstruction

FIGURE: HARDI vs HYDI Sampling.

CONTINUOUS REPRESENTATION OF THE SIGNAL

Continue representation of the MR signal E in the following basis (Spherical Polar Fourier SPF):

$$E(\mathbf{q}) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{nlm} R_n(||\mathbf{q}||) y_l^m \left(\frac{\mathbf{q}}{||\mathbf{q}||} \right) \quad (1)$$

where a_{nlm} expansion coefficients, R_n and y_l^m are respectively are radial and angular atoms.

The basis is *orthonormal* in spherical coordinates:

$$\int_{\mathbf{q} \in \mathbb{R}^3} \left[R_n(||\mathbf{q}||) y_l^m \left(\frac{\mathbf{q}}{||\mathbf{q}||} \right) \right] \cdot \left[R_{n'}(||\mathbf{q}||) y_{l'}^{m'} \left(\frac{\mathbf{q}}{||\mathbf{q}||} \right) \right] d\mathbf{q} = \delta_{nn'} \delta_{ll'} \delta_{mm'} \quad (2)$$

► Skip proof

Any complex function f defined on the sphere can be described by a Spherical Harmonic series:

$$f(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{lm} Y_l^m(\theta, \phi) \quad (3)$$

The diffusion signal is real and symmetric

⇒ Use of a real and symmetric subset

- robust estimation
- reduction of coefficients number

$$y_l^m = \begin{cases} \sqrt{2} \operatorname{Re}(Y_l^m) & \text{if } -l \leq m < 0 \\ Y_l^0, & \text{if } m = 0 \\ \sqrt{2} \operatorname{Im}(Y_l^m) & \text{if } 0 < m \leq l \end{cases} \quad (4)$$

PROBLEM

Which atom is adequate for the radial signal attenuation ?

PROPOSITION

We propose the use of Gauss-Laguerre functions:

$$R_n(\|\mathbf{q}\|) = \left[\frac{2}{\gamma^{3/2}} \frac{n!}{\Gamma(n + 3/2)} \right]^{1/2} \exp\left(-\frac{\|\mathbf{q}\|^2}{2\gamma}\right) L_n^{1/2}\left(\frac{\|\mathbf{q}\|^2}{\gamma}\right), \quad (5)$$

where γ is the scale factor.

Let L_n^k be a generalized Laguerre polynomial:

$$L_n^k = \frac{1}{n!} \sum_{i=0}^n \frac{n!}{i!} \binom{k+n}{n-i} (-x)^i \quad (6)$$

The Gauss attenuation comes from the normalization of these polynomials:

$$\int_0^\infty e^{-x} x^k L_n^k(x) L_{n'}^k(x) dx = \frac{(n+k)!}{n!} \delta_{nn'} \quad (7)$$

RADIAL ATOMS

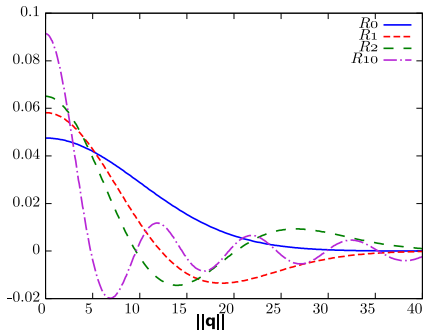


FIGURE: Some radial atoms R_n ,
 $\gamma = 100$

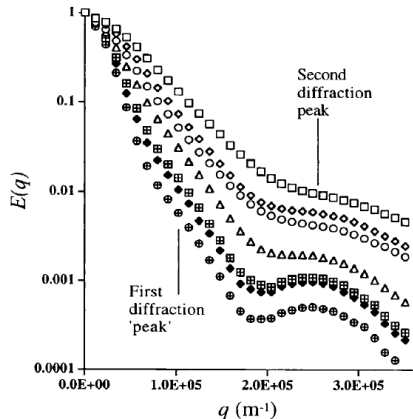


FIGURE: Experimental plot
[Kuchel97]

» Skip estimation

ESTIMATION METHOD

Let M be the basis matrix:

$$\tilde{M} = \begin{bmatrix} R_0(\|\mathbf{q}_1\|)y_0^0\left(\frac{\mathbf{q}_1}{\|\mathbf{q}_1\|}\right) & \dots & R_N(\|\mathbf{q}_1\|)y_L^L\left(\frac{\mathbf{q}_1}{\|\mathbf{q}_1\|}\right) \\ \vdots & \ddots & \vdots \\ R_0(\|\mathbf{q}_{n_s}\|)y_0^0\left(\frac{\mathbf{q}_{n_s}}{\|\mathbf{q}_{n_s}\|}\right) & \dots & R_N(\|\mathbf{q}_{n_s}\|)y_L^L\left(\frac{\mathbf{q}_{n_s}}{\|\mathbf{q}_{n_s}\|}\right) \end{bmatrix}, \quad (8)$$

Let E be the MR signal vector:

$$\tilde{E} = (E(\mathbf{q}_1), \dots, E(\mathbf{q}_{n_s}))^T \quad (9)$$

Let A be the coefficient vector:

$$\tilde{A} = (a_{000}, \dots, a_{NLL})^T \quad (10)$$

The coefficient estimation is computed by a Damped Least Square

$$\tilde{A} = \arg \min_{\tilde{A}} \|\tilde{E} - \tilde{M}\tilde{A}\|^2 = (\tilde{M}^T \tilde{M} + \lambda_l \tilde{L} + \lambda_n \tilde{N})^{-1} \tilde{M}^T \tilde{E} \quad (11)$$

COMPUTATION OF A CHARACTERISTIC ON THE PDF

We have a continuous representation $\hat{E}$ of the raw signal E .

What can be computed on the PDF ?

$\Rightarrow$ Let $\mathcal{G}$ be a characteristic on the PDF at point $\mathbf{k}$:

$$\mathcal{G}(\mathbf{k}) = \int P(\mathbf{p}) h_{\mathbf{k}}(\mathbf{p}) d\mathbf{p} \quad (12)$$

where $h_{\mathbf{k}}$ is a projection function at point $\mathbf{k}$.

How to compute a characteristic on the PDF ?

$\Rightarrow \hat{E} + \text{FFT}[\hat{E}] + \text{Projection Eq.(12)}$

PROBLÈME

Voluminous set of data $\Rightarrow$ Heavy computations

PROPOSITION

Projection in SPF basis.

$$\mathcal{G}(\mathbf{k}) = \int_{\mathbf{p} \in \mathbb{R}^3} P(\mathbf{p}) h_{\mathbf{k}}(\mathbf{p}) d\mathbf{p} = \int_{\mathbf{q} \in \mathbb{R}^3} E(\mathbf{q}) H_{\mathbf{k}}(\mathbf{q}) d\mathbf{q} = \sum_{nlm}^{\infty} a_{nlm} b_{nlm}^{\mathbf{k}} \quad (13)$$

where

- $H_{\mathbf{k}}$ FFT of $h_{\mathbf{k}}$
- $a_{nlm}, b_{nlm}^{\mathbf{k}}$ are respectively the SPF expansions of E and $H_{\mathbf{k}}$.

$\Rightarrow$ The computation of $\mathcal{G}(\mathbf{k})$ is simply a dot product between two SPF coefficients vectors.

$\mathcal{G}$	ODF	FRT	ISO
$h_{\mathbf{k}}(\mathbf{p})$	$\delta(1 - \frac{ \mathbf{p} \cdot \mathbf{k} }{\ \mathbf{p}\ \ \mathbf{k}\ })$	$J_0(2\pi \ \mathbf{q}'\ \ \mathbf{p}\)$	$\delta(\mathbf{k})$

TABLE: A non exhaustive list of PDF characteristics $\mathcal{G}$ and their associated projection function $h_{\mathbf{k}}$ at point $\mathbf{k}$.

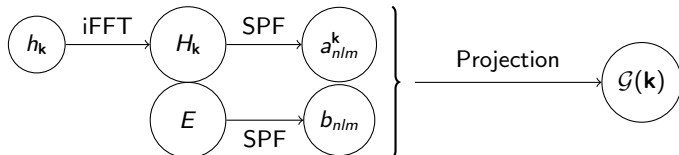


FIGURE: Overview of the algorithm for the computation of a PDF characteristic $\mathcal{G}$ at point $\mathbf{k}$

The coefficients $a_{nlm}^{\mathbf{k}}$ have to be computed only once. $\Rightarrow$ Storage of $a_{nlm}^{\mathbf{k}}$ for all points $\mathbf{k}$.

[» Skip optimization](#)

CHARACTERISTICS ON THE SPHERE $\mathcal{S}^2$

An alternative faster scheme for the characteristics defined on the sphere.

Indeed, the spherical harmonic rotation can be written as:

$$\hat{R}(\alpha, \beta, \gamma) \cdot y_l^m(\theta, \phi) = \sum_{m'} y_l^{m'}(\theta, \phi) \cdot R_{mm'}^{(l)}(\alpha, \beta, \gamma), \quad (14)$$

where $\hat{R}$ stands for the rotation operator in Euler angles. Let a_{nlm} and a'_{nlm} be a linear SPF combinaison and its rotation. So we can write:

$$a'_{nlm} = \sum_{m'=-l}^l a_{nlm'} R_{mm'}^{(l)}(\alpha, \beta, \gamma), \quad (15)$$

where $R_{mm'}^{(l)}$ is the real Wigner rotation matrix.

The rotation of a whole volume composed of numerous voxels can be done by a few operations on its SPF coefficients.

ALGORITHM ON THE SPHERE $\mathcal{S}^2$

Integrating the rotation into estimation method

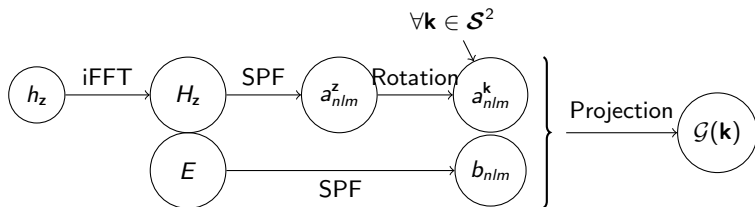
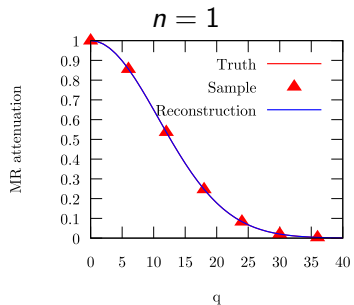
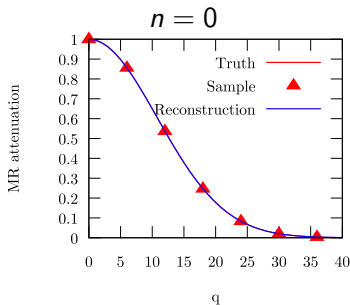
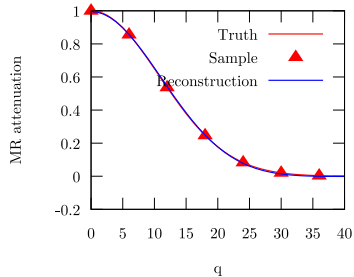
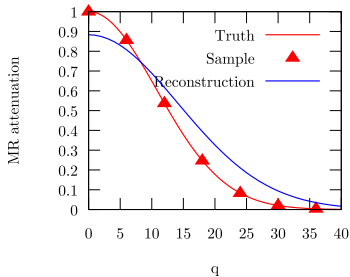


FIGURE: Overview of the algorithm for the computation of a PDF characteristic $\mathcal{G}$ defined on the sphere $\mathcal{S}^2$ at point $\mathbf{k}$.

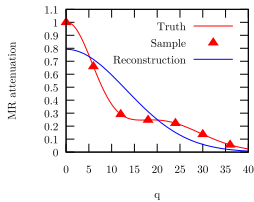
RESULTS: RADIAL RECONSTRUCTION



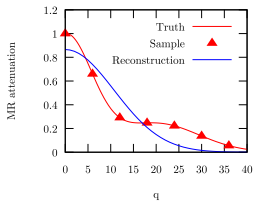
$n = 2$

$n = 3$

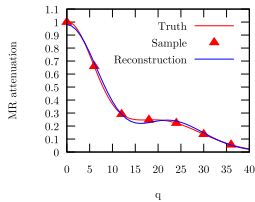
RESULTS: RADIAL RECONSTRUCTION



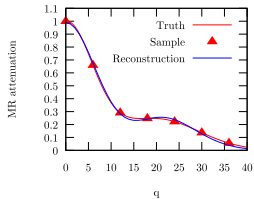
$n = 0$



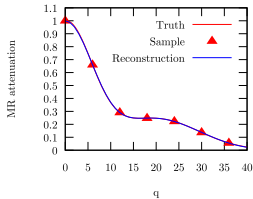
$n = 1$



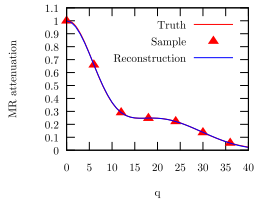
$n = 2$



$n = 3$

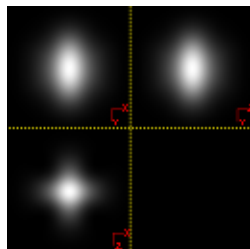
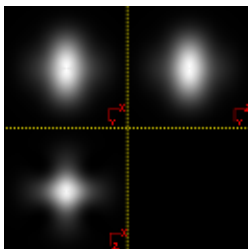
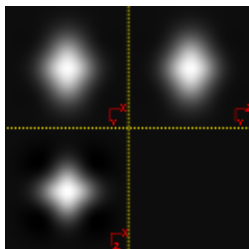
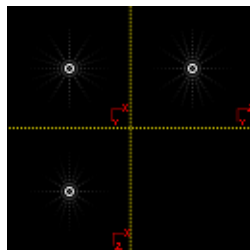
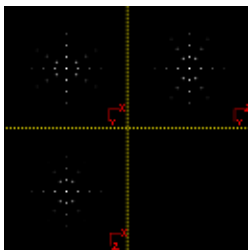
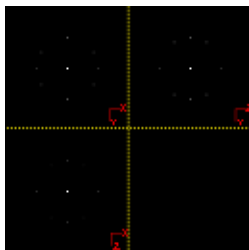


$n = 4$



$n = 5$

RESULTS: RECONSTRUCTION VS SAMPLES



$N = 1, L = 4, \gamma = 100$
1 sphere - 42 directions
PSNR: 33.337902
30 Coefficients

$N = 4, L = 4, \gamma = 70$
3 spheres - 42 directions
PSNR: 45.172752
45 Coefficients

$N = 6, L = 6, \gamma = 50$
10 spheres - 162 directions
PSNR: 50.255381
168 Coefficients

SYNTHETICAL RESULTS: ODF

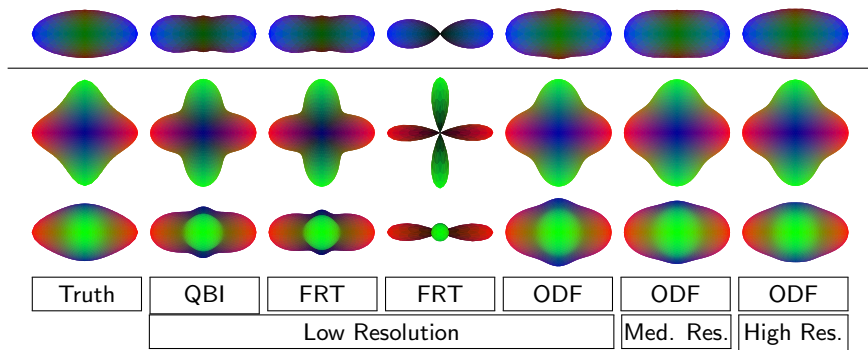
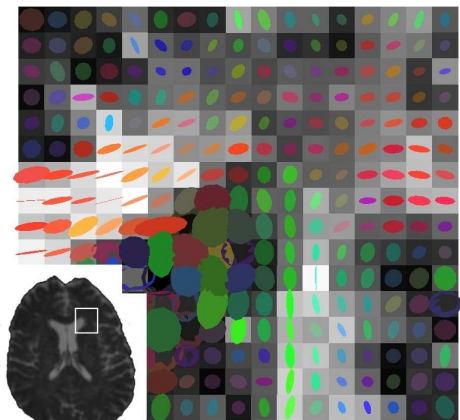


FIGURE: ODF comparisons.

1st line: single fiber.

2nd, 3rd lines: crossing fibers (face and profile view)

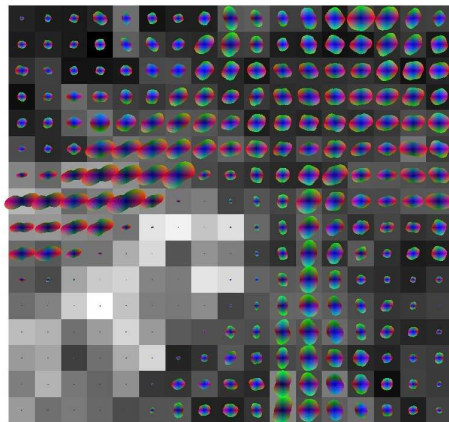
RESULTS OF *in-vivo* EXPERIENCES



(a) DTI (Basser94)

FIGURE: Brain white matter: ODF overlaid on a GFA map. DTI(a) and QBI(b) were computed with $b = 3000 \text{ s/mm}^2$. Our method (c) shows the obtained ODF with $b = 1000$ and 3000 s/mm^2 .

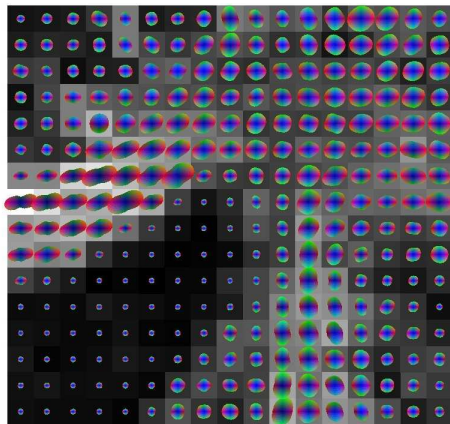
RESULTS OF *in-vivo* EXPERIENCES



(b) QBI (Descoteaux07)

FIGURE: Brain white matter: ODF overlayed on a GFA map. DTI(a) and QBI(b) were computed with $b = 3000 \text{ s/mm}^2$. Our method (c) shows the obtained ODF with $b = 1000$ and 3000 s/mm^2 .

RESULTS OF *in-vivo* EXPERIENCES



(c) Our method, $\mathcal{G}=\text{ODF}$

FIGURE: Brain white matter: ODF overlayed on a GFA map. DTI(a) and QBI(b) were computed with $b = 3000 \text{ s/mm}^2$. Our method (c) shows the obtained ODF with $b = 1000$ and 3000 s/mm^2 .

RESULTS OF *in-vivo* EXPERIENCES

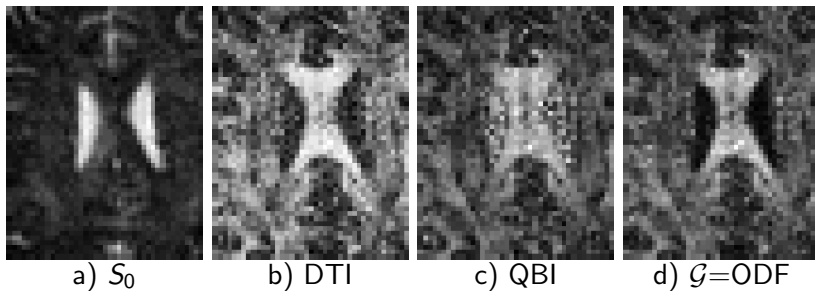


FIGURE: Comparison of GFA [Tuch99] in regions surrounding the corpus callosum and the lateral ventricle. Isotropic area are black, anisotropic area are white.

- Direct method for the estimation of *various* characteristics on tissue microstructure from MR images.
- *Flexible* approach: The more the number of samples, the less the a-prior on signal.
- Concerning the ODF: competing results with states of the arts methods. Differences on cerebro-spinal liquid.

Thank you for your attention.