

Combinatorial Pyramids :

Pyramid of Combinatorial maps

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Combinatorial Pyramids



belongs to the Irregular pyramids family :
Stack of successively reduced graphs.

Advantages:

- Reduction operations may be performed in parallel,
- multi-level description of a same object (e.g. image)

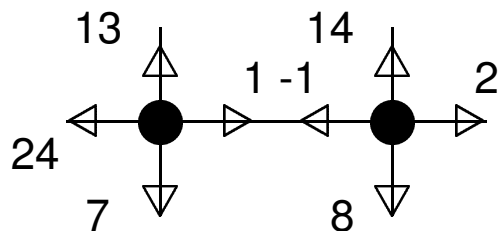


Content

- Combinatorial maps,
- Construction of a pyramid,
- Encoding of the pyramid : implicit,
- Conclusion.



Combinatorial maps



$$G = (\mathcal{D}, \sigma, \alpha)$$

$$\alpha = (1, -1) \dots$$

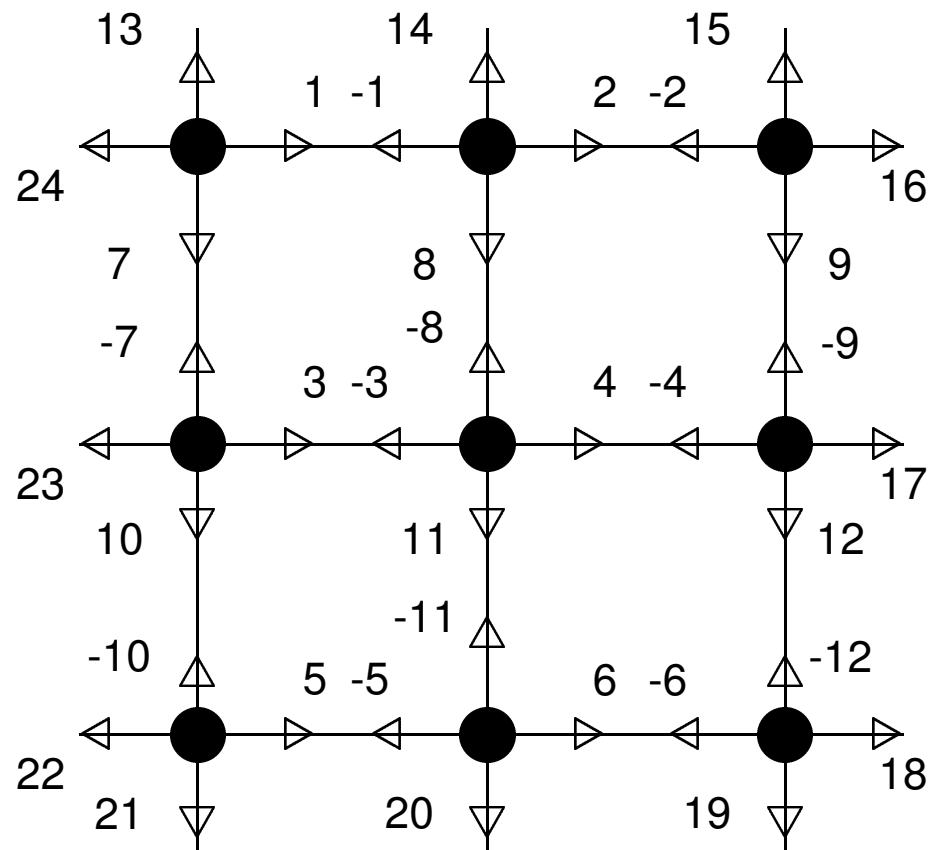
$$\sigma = (13, 24, 7, 1)(14, -1, 8, 2) \dots$$

Combinatorial maps properties



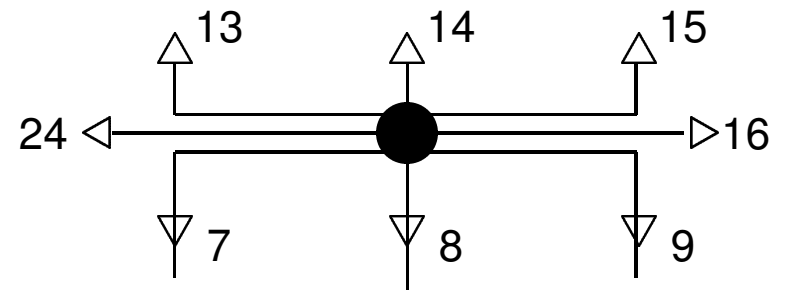
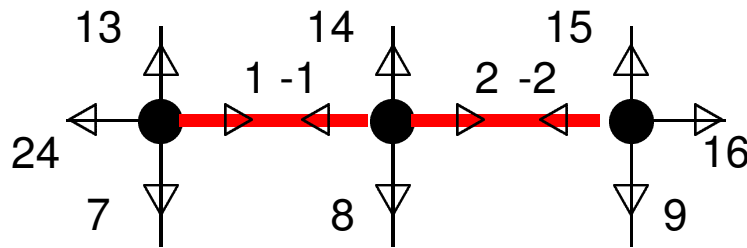
1. One edge for each boundary between two regions,
2. Encoding of inclusions relationships,
3. Implicit encoding of the dual graph,
4. Ordering of the boundaries around each region (σ).

A 3×3 grid



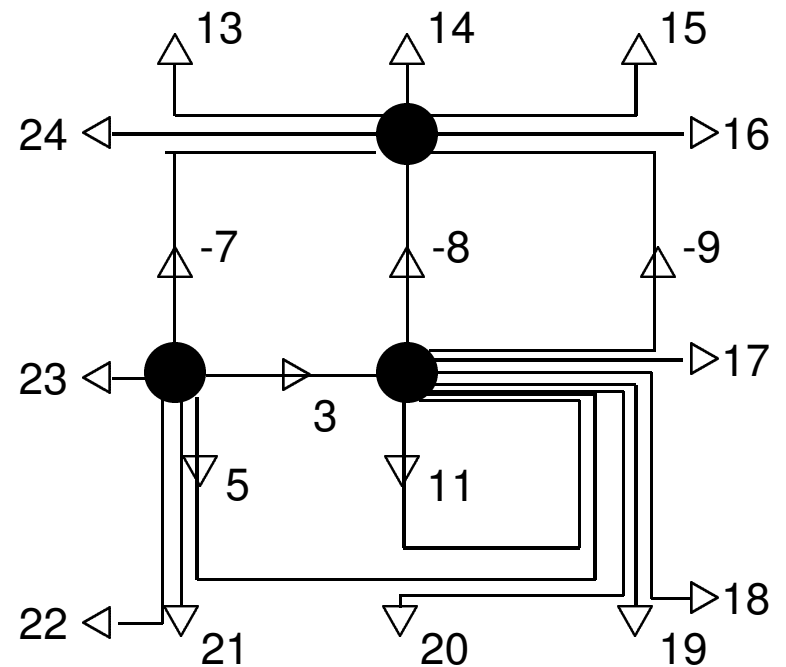
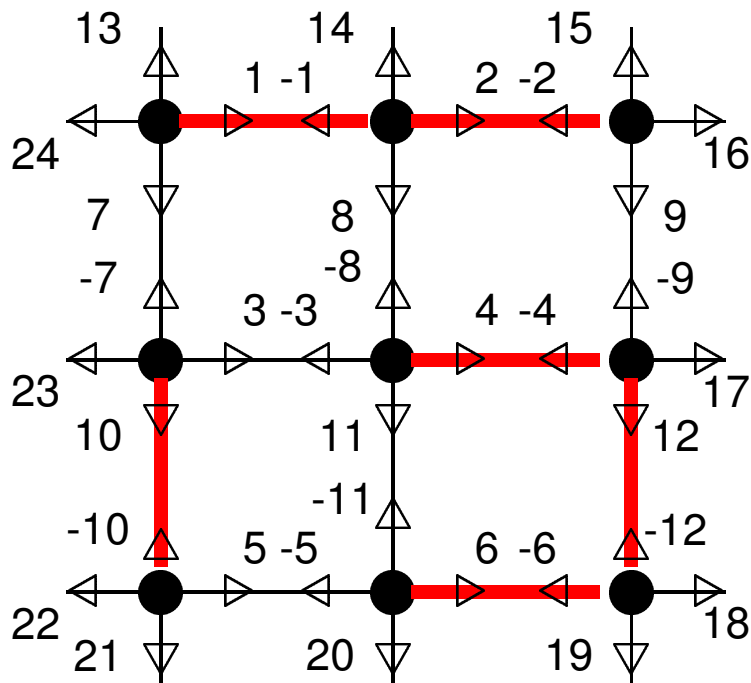
$$G = (\mathcal{D}, \sigma, \alpha)$$

Reduction: Contraction



Contraction Kernel : Forest of the graph

Reduction : Contraction kernel



Reduced Level



$$G = (\mathcal{D}, \sigma, \alpha) \xrightarrow{K} G' = (\mathcal{SD} = \mathcal{D} - K, \sigma', \alpha)$$

$$\forall d \in \mathcal{SD} \quad \sigma'(d) = \varphi^n(\alpha(d))$$

with $n = \text{Min}\{ k \in \mathbf{N}^* \mid \varphi^k(\alpha(d)) \in \mathcal{SD} \}$ and
 $\varphi = \sigma \circ \alpha$

Reduction Window :

$$RW(d) = d.\varphi(\alpha(d)) \dots \varphi^{n-1}(\alpha(d))$$

Construction of the pyramid

 G_i 

contraction kernel

 G_{i+1} 

removal kernel

 G_{i+2}

$$\mathcal{P} = (G_0, G_1 \dots, G_n).$$

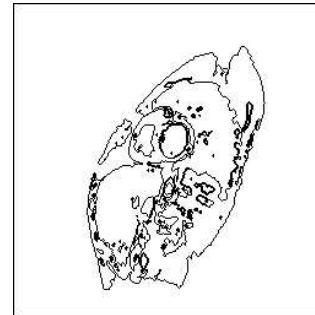
Example



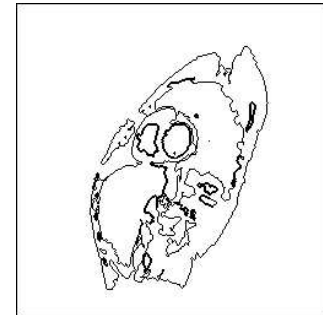
original



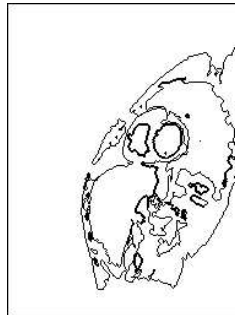
level 1



level 2



level 3



level 4

Embedding : receptive field (recursive uses of the reduction window's parent-child relationship).

A new encoding scheme



$\mathcal{P} = (G_0, G_1 \dots, G_n)$ with $G_i = (\mathcal{D}_i, \sigma_i, \alpha)$ and :

$$\mathcal{D}_n \subset \mathcal{D}_{n-1} \cdots \subset \mathcal{D}_0$$

Two functions :

$$\begin{cases} \forall b \in \mathcal{D}_0 & b \in \mathcal{D}_{level(b)-1}, b \notin \mathcal{D}_{level(b)} \\ \forall i \in \{1, \dots, n\} & state(i) \in \{Contracted, Removed\} \end{cases}$$

$$\rightarrow \mathcal{P} = (G_0, level, state)$$

Construction scheme : $\mathcal{P} = (G_0, G_n, level)$

Retrieving features

■ Bottom-up

	sequential	parallel
G_i	$\mathcal{O}(\partial G_i)$	$\mathcal{O}(\log(\partial G_i))$
$(G_1, \dots, G_n)$	$\mathcal{O}(\mathcal{D}_0)$	$\mathcal{O}(\log(\mathcal{D}_0))$

■ Top-down:

		sequential	parallel
<i>region</i>	$\sigma_i^*(b)$	$\mathcal{O}(R_{\sigma_i^*(b)})$	$\mathcal{O}(\log(R_{\sigma_i^*(b)}))$
<i>border</i>	b	$\mathcal{O}(\partial RF_i(b))$	$\mathcal{O}(\log(\partial RF_i(b)))$



Backup of the Pyramid

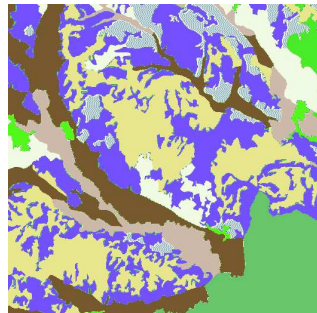


Storage of $level()$ and $state()$ in $\overline{G}$.

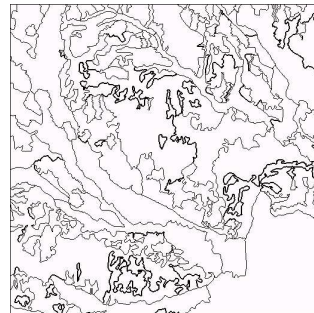
If G encodes a $n \times m$ grid

→ $(n + 1) \times (m + 1)$ 24 bits color image.

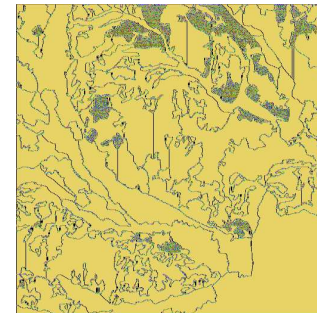
→ 1365 partitions.



original



top level



backup

Conclusion



Q: Why should I use Combinatorial Pyramids ?

Re: Why not. 😊

- Explicit encoding of the orientation,
- Implicit encoding of the dual graph,
- Explicit or implicit encoding of the pyramid.

GPL C & C++ code :

www.univ-reims.fr/leri/membre/luc/PYRAMIDES/index.html