Mathematical Morphology

Opening - Closing

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Set Opening

The set opening of X by the structuring element B is defined as :

$$\gamma_B(X) = \delta_{\breve{B}}(\mathcal{E}_B(X))$$

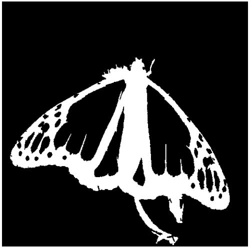
We have :

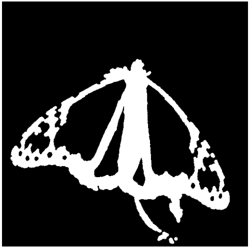
$$\gamma_B(X) = \bigcup_{x \in E} \{B_x \mid B_x \subseteq X\}$$

If B is symmetric we obtain :

$$\gamma_B(X) = \delta_B(\mathcal{E}_B(X))$$

Example :





Set opening : properties

The opening operator :

- Preserve the shape of objects,
- remove thin parts or small objects,
- may divise objects

Anti-exensivity :

$$\gamma_B(X) \subseteq X,$$

Growth :

$$X \subset Y \Rightarrow \gamma_B(X) \subset \gamma_B(Y),$$

Idempotence :

$$\gamma_B(\gamma_B(X)) = \gamma_B(X).$$

Topological property :

- Do not preserve the connexity,
- non homotopic transformation,

Function opening

The opening of a real function f by a structuring element B is defined by :

$$\gamma_B(f) = \delta_{\check{B}}(\mathcal{E}_B(f))$$

We have (anti-extensivity property) :

$$\gamma_B(f) \leq f$$

Example :

original — radius 3 — radius 6

Set closing operator

The closing of a set X by a structuring element B is defined as :

$$\varphi_B(X) = \mathcal{E}_{\check{B}}(\delta_B(X))$$

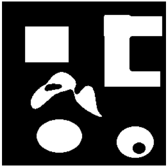
The closing of a set is equal to the complementary of the opening of its complementary :

$$\varphi_B(X) = \mathcal{C}_E(\gamma_B(\mathcal{C}_E(X)))$$

Hence for any x, if $B_x \subset \mathcal{C}_E(X)$, then B_x is included in the complementary of $\varphi_B(X)$ in E.

Example :

original



radius 3

radius 6

radius 12

Set closing : properties

The closing operator :

- Fill small cavities,
- connect close connected components.

Extensivity : $X \subset \varphi_B(X)$,

Growth :

$$X \subset Y \Rightarrow \varphi_B(X) \subset \varphi_B(Y)$$

Idempotence :

$$\varphi_B(\varphi_B(X)) = \varphi_B(X)$$

Continuity : Semi continuous superiourly

Topology :

- do not preserve the continuity,
- non homotopic.

Function closing

The closing of a real function f by a structuring element B is defined by :

$$\varphi_B(f) = \mathcal{E}_{\breve{B}}(\delta_B(f))$$

A closing operator removes small local minimum while preserving the highest values. The image is globally lighter.
Examples :

original | radius 3 | radius 6